

Research on Tracing the Cycle Force Value of Fatigue Testing Machine Based on Linear Vibration Mechanics Model

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Abstract: This study proposes a novel correction method that differs from the JJG556-2011 standard. By establishing an undamped three-degree-of-freedom (3-DOF) vibration mechanics model, the proposed method eliminates the need for measuring dynamic inertial displacement. Instead, the correction factor is derived analytically based on the measured inertial mass, the stiffness of the fatigue testing machine's force sensor (i.e., the stiffness between the lower fixture and the base), and the operating frequency, thereby improving correction accuracy. Furthermore, examining a locally equivalent damped singledegree-of-freedom (SDOF) linear vibration model, it was found that the difference between the correction factors obtained with and without damping consideration is merely 0.08%. Consequently, calibration results can be effectively corrected using the correction factor derived under undamped conditions without significant loss of precision.

Keywords: Cyclic Force Calibration; Correction Factor; Fatigue Testing Machine

1. Introduction

Fatigue testing machines are mainly used to study the fatigue performance of various materials in industries such as aerospace, automotive and shipbuilding, and construction engineering under cyclic forces. They can also be used for pre fabricated crack and crack propagation tests, including electro-hydraulic servo fatigue testing machines, hydraulic pulsation fatigue testing machines, mechanical fatigue testing machines, and electromagnetic resonance fatigue testing machines.^[1~5] Due to the inadequate theoretical and methodological framework for dynamic measurement and calibration of cyclic forces in fatigue testing machines, the calibration of force values for

fatigue testing machines is still commonly conducted under static conditions in China. [6~7] With the widespread use of fatigue testing machines, especially the 50 ton high-frequency resonant fatigue testing machine, which has appeared in domestic material fatigue testing through independent research and development, there is an urgent need for research and development on the dynamic measurement and calibration of the cyclic force of fatigue testing machines.

2. Regarding the Inertial Force Correction in Dynamic Calibration of High Frequency Fatigue Testing Machines

According to JJG556-2011[8], the cyclic force calibration of fatigue testing machines is mainly carried out using standard force gauges or resistance strain gauges with natural frequencies not lower than 15 times the maximum operating frequency of the tested fatigue testing machine. By connecting the cyclic force calibration force sensor in series with the fatigue testing machine force sensor, the fatigue testing machine outputs an alternating load of sine wave. By collecting the cyclic force amplitude and peak value of the fatigue testing machine and comparing them with the standard value collected by the force calibration device, the indication error and repeatability of the cyclic force amplitude and peak value can be obtained.

In the calibration process of dynamic forces, due to the inertia force of large mass blocks, according to JJG556-2011, the calibration results of cyclic forces should be corrected for inertia force. The regulation provides the calculation formula for inertia force $F_i = -m(2\pi f)^2 X$, M is the inertial mass; F is the operating frequency of the fatigue testing machine; X is the displacement of the inertial mass.

For high-frequency and high load electromagnetic resonance fatigue testing machines, due to the



large test force value, the mass of the upper and lower fixtures is usually large. When performing cyclic force calibration, the mass of the lower fixture will generate an inertial force that cannot be ignored, resulting in the force sensor of the cyclic force calibration device and the force sensor of the fatigue testing machine not being subjected to the same force. Taking the GPS500 high-frequency fatigue testing machine produced by SINOTEST as an example, the mass of the lower fixture is 306kg. Under the working conditions of a peak cyclic force of 30kN and a cyclic force range of 20kN, the working frequency of the fatigue testing machine reaches 140.6Hz, and the inertial force of the lower fixture reaches 2.08kN. The calibration of the cyclic force under these conditions must consider the correction of the inertial force of the lower fixture. However, according to JJG556-2011, in order to achieve the correction of inertial force, it is necessary to actually measure the dynamic inertial mass displacement of the fixture. According to calculations, this displacement is only 0.0087mm, and it is difficult to obtain dynamic displacement measurement results with an uncertainty of less than 1% through actual measurement under the condition of a frequency of up to 140.6Hz. This cannot meet the accurate correction of the relative error of 3% in the indication of the high-frequency resonant fatigue testing machine. Therefore, it is necessary to search for a new correction formula to achieve the correction of inertial force.

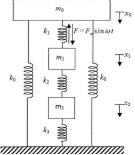


Figure 1. Three Degree of Freedom Linear Vibration Model

3. Deduction of Cyclic Force Correction Factor based on Linear Vibration Mechanics Model

When the fatigue testing machine is used for fatigue testing or cyclic force calibration, the system actually exists under damping conditions. After actual measurement, the damping coefficient of the system under rigid connection with the standard force sensor is 0.05. For the sake of simplicity in derivation, we have established a three degree of freedom linear vibration

mechanics model without damping.

Establish the motion equation of the system:

$$m_0\ddot{x}_0 + 2k_0x_0 + k_1(x_0 - x_1) = -F_m \sin \omega t$$
 (1)

$$m_1\ddot{x}_1 + k_1(x_1 - x_0) + k_2(x_1 - x_2) = F_m \sin \alpha t$$
 (2)

$$m_2\ddot{x}_2 + k_2(x_2 - x_1) + k_3x_2 = 0$$
 (3)

$$m_{2}x_{2} + k_{2}(x_{2} - x_{1}) + k_{3}x_{2} = 0$$
The steady-state solution is:
$$x_{0} = \frac{-(m_{1}\alpha^{2} - k_{2})(m_{2}\alpha^{2} - k_{3}) - m_{1}k_{2}\alpha^{2}}{D} F_{m} \sin \alpha t$$
(4)

$$x_1 = \frac{-(m_0\omega^2 - k_0')(-m_2\omega^2 + k_2 + k_3)}{D} F_m \sin \omega t$$
 (5)

$$x_{2} = \frac{-k_{2}(m_{0}\omega^{2} - k_{0}')}{D} F_{m} \sin \omega t$$
 (6)

Due to

$$F_{FatigueTes\,ter} = k_3 x_2 \tag{7}$$

$$F_{reference} = k_2(x_2 - x_1) \tag{8}$$

The correction factor β is

$$\beta = \frac{F_{Fatiguetester}}{F_{reference}} = \frac{k_3}{k_3 - m_2 \omega^2} \tag{9}$$

Of which, $F_{Fatigue Tester}$ is the cyclic force value of

the fatigue testing machine force sensor; $F_{reference}$ is the cyclic force value of the force sensor in the cyclic force calibration device.

Through theoretical deduction, it has been found that due to the inertia force of the lower fixture in the system, the force sensor of the fatigue testing machine is always subjected to a force greater than that of the force sensor of the cyclic force calibration device. That is to say, when calibrating the cyclic force of the fatigue testing machine, at the same time, the real-time cyclic force displayed by the fatigue testing machine is higher than the cyclic force measured in real-time by the calibration device. The difference between the two cannot be simply considered as the indication error of the dynamic measurement of the force sensor of the fatigue testing machine, but should be corrected by considering the correction factor for its deviation source before calculation. The correction factor is only related to the mass of the lower fixture (m₂), working frequency, and stiffness of the fatigue testing machine force sensor (k₃) of the system. The measurement of these three parameters is much more convenient and less uncertain than the measurement of inertial mass displacement (X).

4. Considering the Correction Factor of Cyclic **Force under Damping Conditions**

Due to the fact that the system actually exists under damping conditions, although the damping



coefficient of the system has been measured to be 0.05, in order to further analyze the influence of damping on the cyclic force correction factor, the GPS500 high-frequency fatigue testing machine produced by SINOTEST is still used as an example to study the cyclic force correction factor under damping conditions.

In vibration systems, damping significantly affects the elastic recovery force and inertial force in the system. In theory, damping does not directly change the magnitude of elastic recovery force, but damping consumes vibration energy and gradually attenuates the amplitude of the system. Therefore, the peak value of elastic force calculated without considering damping will be higher than the actual value. For inertial forces, damping can significantly suppress the wireless increase of inertial forces under resonance conditions. In addition, under steady-state forced vibration, damping will change the phase difference of the system, causing the phase of the elastic restoring force to lag behind the excitation load. At the same time, under the condition of no damping, the inertial force and elastic restoring force are always opposite, with a phase difference of 180 $^{\circ}$. However, under the condition of damping, the phase difference approaches 180°, and its value is related to the damping ratio.

From the analysis above, it can be seen that the correction factor β is only related to the mass of the lower fixture (m_2) , operating frequency ω , and stiffness of the fatigue testing machine force sensor (k₃) of the system. So, the entire vibration system can be equivalent to a local single degree of freedom linear vibration system, and on this basis, the correction factor considering damping conditions can be studied.

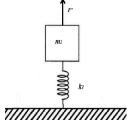


Figure 2. Single Degree of Freedom Linear **Vibration Model**

Equivalent the internal force load of spring k_2 , the cyclic force calibration device in the three degree of freedom linear vibration model, to the external force load F' in the single degree of freedom linear vibration model. At this time, by measuring the high-frequency fatigue testing machine of GPS500 model, the spring stiffness k₃ is

214300000N/m, and the working frequency is 140.6Hz, establish the motion equation of the system:

$$m_2\ddot{x}_2 + c\dot{x}_2 + k_3x_2 = -F'\sin\omega t$$
 (10)

The steady-state solution is

$$x_2(t) = X\sin(\omega t - \varphi) \tag{11}$$

$$X = \frac{\frac{F'}{k_3}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}$$
(12)

$$\varphi = \arctan\left(\frac{2\zeta \frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega}\right)^2}\right)$$
 (13)

Among them, ω_n is the undamped natural frequency of a single degree of freedom system; 5 is the damping ratio of a single degree of freedom

Therefore, considering the damping situation, the correction factor β' is

$$\beta' = \frac{F_{Fatiguetester}}{F_{reference}} = \frac{F' \sin(\omega t)}{k_3 X \sin(\omega t - \varphi)}$$
 (14)

According to JJG556-2011, it is known that the error in the peak value or range of cyclic force indication is only related to the peak value or range of cyclic force indicated by the force indicator device of the fatigue testing machine and the peak value or range of cyclic force indicated by the force calibration device. Therefore, this correction factor can ignore the phase difference between the two, that is

$$\beta' = \frac{F_{Fatiguetester}}{F_{reference}} = \frac{F'}{k_3 X}$$

 $\beta' = \frac{F_{Fatiguetester}}{F_{reference}} = \frac{F'}{k_3 X}$ Therefore, taking the GPS500 high-frequency fatigue testing machine produced by SINOTEST as an example, the correction factors for considering damping and not considering damping under this working condition are calculated in Table 1, and the force value of inertia force is taken as the reference, taking 1. Figures 3 and 4 show the amplitude and phase relationship of each force when the damping coefficient of the vibration system is 0.5 and 0.05, respectively.

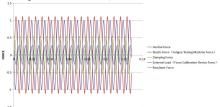


Figure 3. Schematic Diagram of the Amplitude **Phase Relationship of Various Forces** (Damping Coefficient 0.5)





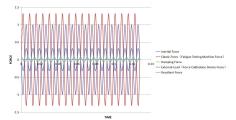


Figure 4. Schematic Diagram of the Amplitude Phase Relationship of Various Forces (Damping Coefficient 0.05)

Table 1. Correction Factor Calculation Result

GPS500 high-frequency fatigue testing machine				
Lower fixture	Stiffness of force	Operating	Damping	Correction
quality(kg)	sensor(N/m)	Frequency(Hz)	coefficient	factor
306	2143000000	140.6	0.5	105.35%
306	2143000000	140.6	0.05	112.46%
306	2143000000	140.6	0	112.54%

It can be seen that considering the correction factor under damping conditions, there is only a difference of 0.08% compared to not considering damping. For a high-frequency resonant fatigue testing machine with a relative error limit of 3% in indication, the influence of damping can be ignored when correcting calibration results.

5. Conclusion

This article takes a high load high-frequency resonant fatigue testing machine as a case study and finds that when correcting the inertial force according to JJG556-2011, there are difficulties in measuring the inertial mass displacement of this type of fatigue machine or the measurement accuracy cannot meet the correction requirements. Therefore, by establishing a three degree of freedom linear vibration mechanics model, a correction factor β is derived that is only related to the mass of the lower fixture (m₂), the working frequency ω , and the stiffness of the fatigue testing machine force sensor (k₃) of the vibration system. This correction factor has the advantages convenient measurement and **lower** which is measurement uncertainty, more conducive to the correction of cyclic force calibration results. In addition, this article also investigated the influence of damping conditions on the correction factor by establishing a local equivalent single degree of freedom linear vibration system. It was found that the correction

factor can ignore the phase difference between the fatigue testing machine force value and the calibration device force value, and considering the correction factor under damping conditions, the difference is only 0.08% compared to the case without considering damping. In the calibration process, the correction factor without considering damping can be directly used, and this uncertainty can meet the error judgment requirements for general fatigue testing machines.

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