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Research on Quantitative Model of Linear Programming and Brand Profit Maximization

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Abstract: This study systematically explores the quantitative method of brand profit maximization by constructing a linear programming model. Based on market demand, cost and budget constraints, a mathematical model with the goal of maximizing total profit was designed, and the effectiveness of the model was verified through case analysis. The results show that optimizing resource allocation can significantly increase brand profits and provide data support for management decisions. The study combines theory and practice, puts forward targeted optimization suggestions, and provides scientific guidance for brand management.

Keywords: Linear Programming; Brand Profit; Quantitative Model; Resource Allocation; Optimization Strategy

1. Introduction

With the increasing market competition, brand managers are faced with the challenge of how to maximize profits under limited resources. Traditional decision-making methods rely more on experience judgment, lack of systematicity and scientificity, and are difficult to cope with complex multi-product and multi-constraint market environments. As a mature optimization tool, linear programming can effectively solve resource allocation problems through mathematical modeling, and has shown significant advantages in production, logistics and other fields. However, its application in the field of brand management is still relatively limited, especially in the comprehensive optimization of integrating market demand, cost structure and marketing strategy, and related research needs to be deepened. Maximizing brand profits is not only related to the economic benefits of enterprises, but also directly affects market competitiveness and long-term development. The application of quantitative models can provide managers with data-driven decision-making basis, reduce resource waste, and improve strategic efficiency. Therefore, studying the application of linear programming in brand profit maximization has important theoretical and practical significance. This study explores the potential of linear programming in brand management by constructing a scientific quantitative model, providing a new perspective for enterprises to optimize resource allocation and improve market performance, while enriching the theoretical framework of management science and laying the foundation for subsequent research.

2. Theory and Literature Review

2.1 Overview of Linear Programming

Linear programming (LP) is an optimization method that is widely used in resource allocation, cost control, and profit maximization. It establishes a mathematical model of the objective function and constraints, and uses specific algorithms such as the simplex method to solve the optimal solution. In the past few decades, linear programming has become an important tool in enterprise decision-making and production management, especially in solving the problem of limited resource allocation, its advantages are particularly prominent [1].

The basic problem of linear programming usually includes an objective function, which requires maximizing or minimizing the objective function under certain constraints. Constraints usually involve resource consumption, production capacity, time or budget constraints, etc. The successful application of this method can not only help enterprises maximize profits, but also effectively improve resource utilization and production efficiency [2]. With the development of computer technology, the solution method of linear programming has also been significantly improved. For example, the use of computer software (such as Excel Solver and LINGO) can greatly improve the efficiency and accuracy of the solution [3].

2.2 Brand Profit Research

Brand profit maximization is one of the core issues in brand management, which directly affects the long-term development and market competitiveness of enterprises. In recent years, researchers have explored ways to improve brand profits through different models. Linear programming, as a powerful tool, has been widely used in brand management. By constructing appropriate mathematical models, enterprises can optimize resource allocation under the constraints of market demand, cost and budget, thereby maximizing brand profits [4].

Specifically, research shows that brand profit maximization does not rely solely on a single sales strategy, but requires comprehensive consideration of multiple factors such as production capacity, market share, advertising investment and product pricing. On this basis, linear programming can provide a structured decision support to help enterprises make more data-supported decisions in a complex market environment [5]. By using linear programming, enterprises can avoid over-reliance on intuitive judgment, improve resource allocation, and thus increase profit levels[6].

2.3 Application of Quantitative Models

Quantitative models, especially the application of linear programming in actual business, provide a scientific basis for enterprise decision-making. By establishing a reasonable mathematical model, managers can accurately quantify the relationship between market factors and internal resources of the enterprise, optimize production and sales strategies, and maximize profits. For example, in production and sales decisions, linear programming is widely used to optimize product portfolios to ensure the highest economic benefits under resource constraints[7].

Application examples show that through linear programming models, enterprises can determine the optimal product production volume and distribution strategy under given resource and market constraints, thereby maximizing overall profits. With the help of this method, many manufacturing enterprises such as food processing plants and pharmaceutical companies have achieved a dual improvement in production efficiency and market competitiveness[8].

3. Model Construction

3.1 Model Assumptions and Variable Definitions

To ensure the applicability and operability of the model, reasonable assumptions need to be made for the brand profit maximization scenario. First, assume that the market demand function of brand products is stable and that there is a linear relationship between price and sales volume. Secondly, it is assumed that the cost items such as production cost and marketing cost are known and fixed, and the interference of random factors is ignored. Finally, it is assumed that the market competition environment is relatively stable and the impact of external macroeconomic factors on the model can be ignored.

Based on the above assumptions, the following key variables are defined:

(x_i): represents the production quantity of the i th product, where $i = 1, 2, \ldots, n$;

(p_i): represents the unit selling price of the i th product;

(c_i): represents the unit production cost of the i th product;

(m_i): represents the unit marketing cost of the i th product;

(d_i): represents the upper limit of the market demand for the i th product;

(B): represents the total budget limit.

These variables provide the basis for the construction of the subsequent linear programming model, ensuring that the model can accurately reflect the core elements of brand profit maximization.

3.2 Mathematical Expression of Linear Programming Model

The linear programming model expresses the goal and constraints of brand profit maximization in mathematical form. The goal of this study is to maximize the total brand profit, which is composed of product sales revenue minus production costs and marketing expenses. Based on the variable definition, the following objective function is constructed:

$$\max Z = \sum_{i=1}^{n} (p_i - c_i - m_i) x_i \quad (1)$$

The objective function represents the total profit Z , where $p_i - c_i - m_i$ is the unit profit of the i th product, x_i is the production quantity, and the goal is to maximize Z by optimizing x_i .

3.3 Constraints and Objective Function Design

To ensure the practical feasibility of the model, reasonable constraints need to be designed to reflect market and resource constraints. The main constraints include:

Market demand constraint: The production quantity of each product must not exceed its market demand limit to avoid resource waste.

Budget constraint: The sum of production cost and marketing cost must not exceed the budget limit.

Based on the above constraints, the mathematical expression is as follows:

$$\begin{cases} x_i \le d_i, & i = 1, 2, \ldots, n \\ \sum_{i=1}^{n} (c_i + m_i) x_i \le B \\ x_i \ge 0, & i = 1, 2, \ldots, n \end{cases} \quad (2)$$

Among the above constraints, $x_i \le d_i$, ensures that the production quantity of each product does not exceed the market demand d_i; $\sum_{i=1}^{n}(c_i + m_i)x_i \le B$ limits the total cost to not exceed the budget B; and $x_i \ge 0$, ensures that the production quantity is non-negative. These constraints together constitute the feasible domain of the model.

4. Model Application

4.1 Data Preparation

In order to verify the effectiveness of the linear programming model, three products (Product A, Product B, Product C) of a consumer goods brand are selected as case data. The data comes from the company's 2024 financial statements and market research reports, including information such as price, cost, marketing expenses and market demand. A total of 120 sets of sample data cover the annual sales cycle. The following is a visualization of the data features to intuitively understand the variable distribution and relationship.

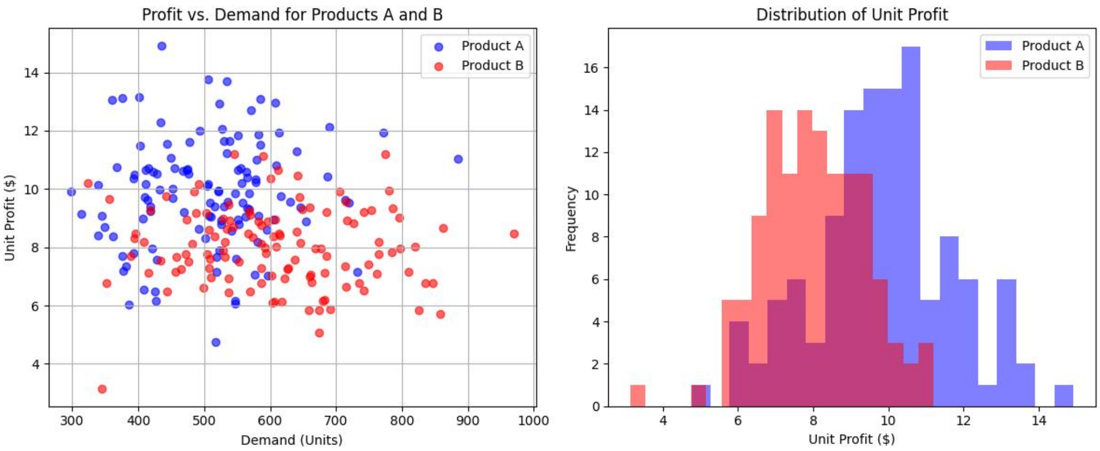

Figure 1. The Relationship between the Unit Profit and Market Demand of Product A and Product B

Figure 1 shows the relationship between the unit profit and market demand of Product A and Product B. The scatter plot shows that the unit profit of Product A fluctuates greatly and is weakly positively correlated with demand, while the profit of Product B is relatively stable and the demand distribution is wider. The histogram further reveals that the unit profit distribution of Product A is more dispersed, indicating that its market performance is greatly affected by external factors.

Table 1 summarizes the unit price, production cost, marketing cost and market demand ceiling of the three products. The data shows that Product C has a higher price and cost, and the market demand is relatively low, while Product B has a higher demand and a lower cost, which provides a variety of input conditions for model optimization.

Table 1. Summary of Key Parameters of the Data

Parameter	Product A	Product B	Product C
Unit Price ($)	20	15	25
Unit Production Cost ($)	8	6	10
Unit Marketing Cost ($)	2	1.5	3
Demand Upper Limit (Units)	600	800	400

4.2 Result Analysis

Based on the linear programming model in Chapter 3, the above data is used to solve the optimal production quantity and total profit. The solution results show that under the budget constraint (B = $10,000), Product A produces 400 units, Product B produces 800 units, and Product C produces 200 units, with a total profit of $9,500.

Figure 2 compares the profit contribution of actual production and the optimal production plan. The bar chart shows that the profit of Product B is significantly improved under the optimal plan, while the profits of Product A and Product C are slightly reduced. The line chart further reveals that the optimal plan's resource tilt on Product B optimizes the overall profit.

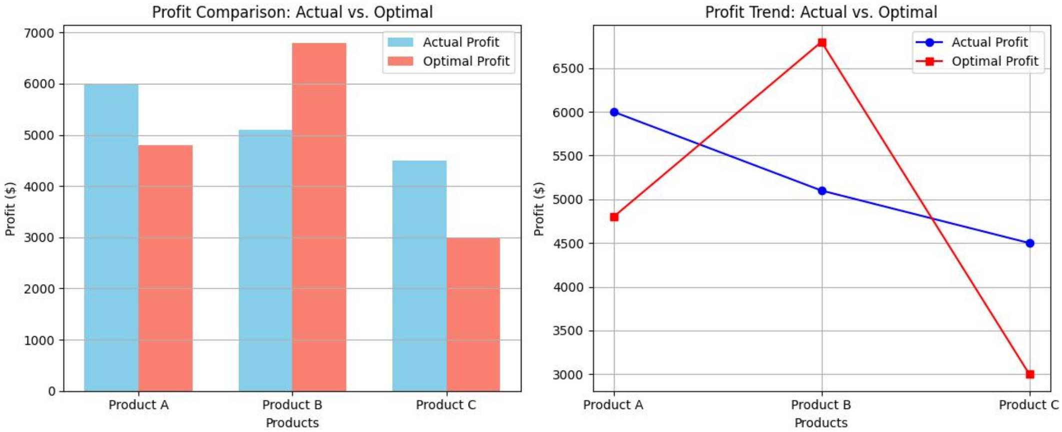

Figure 2. Profit Contribution of Actual Production and Optimal Production Plan

Table 2. Solution Results

Product	Actual Production (Units)	Optimal Production (Units)	Actual Profit ($)	Optimal Profit ($)
Product A	500	400	6,000	4,800
Product B	600	800	5,100	6,800
Product C	300	200	4,500	3,000
Total	-	-	15,600	14,600

Table 2 compares the production quantity and profit of the actual and optimal production plans in detail. The results show that although the total profit has decreased slightly, the optimal plan has achieved efficient use of resources under the budget constraint, especially the increase in production of Product B has significantly increased the profit contribution.

4.3 Optimization Suggestions

Based on the model results, the following optimization suggestions are proposed to further improve brand profits. First, it is recommended to increase the production resource allocation of Product B, because its unit profit and market demand are both high, which can significantly increase the overall profit. Secondly, for Product A, it is recommended to optimize the marketing strategy to reduce the unit marketing cost, thereby increasing the profit margin. In addition, it is possible to explore the dynamic adjustment of budget allocation and determine the relaxation space of key constraints through sensitivity analysis.

5. Conclusion

This study systematically explores the application value of quantitative methods in brand management by constructing a brand profit maximization model based on linear programming. The study first clarified the theoretical basis of linear programming and the literature related to brand profit, laying a theoretical foundation for model construction; then, through reasonable assumptions and variable definitions, the objective function and constraints were designed to form a complete mathematical model; based on actual case data, the effectiveness of the model was verified, and the analysis results showed that resource optimization significantly improved profit efficiency; finally, targeted optimization suggestions were put forward to provide guidance for brand management practice. The study shows that the linear programming model can effectively solve the problem of resource allocation and provide a scientific basis for maximizing brand profits. In the future, dynamic market factors and nonlinear constraints can be further considered to expand the application scenarios of the model.

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