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The Formation and Evolution of the Mixed Airline Network

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Abstract: Air transport with respect to the most important advantages of other modes of transport is fast. In order to keep the speed advantage, the airline can gather more, through the construction of hub airport transport, traffic diversion in the network, to ensure aviation network operation smoothly. But with the rapid development of China's economy, high-speed rail and other transportation the way of competitive air transportation is more and more strong, requiring passengers to travel time is more and more high, and the costly process of construction of hub airports, airlines are more willing to choose to open non hub airport between direct flights to take part OD of traffic flow, reduce the load capacity of hub airports. In other words, in the airline network strictly distribution, with the gradual increase of keeping the speed advantage, the airline will choose to open direct flights in the non hub airport, the airline network structure will evolve from strict single allocation hub airline network for mixed airline network. Through the research on the formation and evolution of the mixed airline network, draw the following conclusions: the evolution of mixed airline network, but attention should be paid to hub airport congestion, should also consider different routes in the network change of passenger traffic; when the passenger traffic growth in the a certain percentage range, by opening a new direct route to solve the congestion problem of hub airports; when the passenger traffic growth exceeds a certain proportion, only increase the design capacity of the airport hub or the expansion of the new hub airport in order to ensure the orderly operation of the whole network.

Keywords: Airline Network; Airline Structure; Evolution; Optimization Design

1. Introduction

In the real network, when airlines choose to strictly allocate hub route networks, with the increasing demand for air transportation, the passenger flow of each route will also increase. When the number of hub airports in the network remains unchanged, the maximum capacity (design capacity) that hub airports can bear remains unchanged, and the actual passenger flow through hub airports reaches a certain number, hub node congestion will occur, which will affect the transportation efficiency of the entire route network and the operating revenue of airlines.

There have been many studies on the design of airline network both domestically and internationally. Chen et al. in order to discuss the influence of different subsidy modes on the airline networks of multi-airport systems, constructing a dynamic game model between airports, airlines, and passengers. The Nash equilibrium is obtained by analyzing the payoff matrix [1]. Gu and Li based on multiple factors to maximize the company's benefits, comprehensively considering multiple influencing factors [2]. Jing et al. thought that the Stackelberg dynamic game theory is used to solve the problem， and the uncertainty effect, cost effect and market scale effect of game equilibrium are discussed deeply [3]. He et al. in order to enhance the interaction and muli-view collaboration, the route network structure visualization analysis system based on data flow model is constructed for method validation [4]. Hu et al. in order to improve the robustness of airline network ， the node importance identification of airline network is studied [5]. Hsu and Wen. used the grey theory research method to predict the passenger flow of airlines, and achieved good results in determining flight frequencies and optimizing network structures for airlines under uncertain conditions [6]. Ebery studied the design problem of large-scale hub airline networks [7]. Wojahn used density economy as a calculation basis to design a minimum cost network that combines urban air route networks with single allocation hub air route networks [8]. Skorin Kapov D used the theory of cooperative game theory to establish an attractive cost allocation method and studied the cost allocation problem of non strict hub route networks [9]. O'Kelly and Bryan applied the theory of economies of scale discounts and utilized robust optimization techniques to design hub route networks [10]. Podnar et al. introduced the concept of cost discounts and applied the theory of economies of scale to study the issue of cities transitioning from airline network structures to hub airline network structures [11]. Bertsimas and Sim applied discrete robust optimization methods in the study of airline networks [12]. Elhedhli and Hu considered airport congestion when establishing a hub airline network model [13]. Xu et al. pointed out that in order to achieve the goal of maximizing profits, airlines will adjust their route network structure from the original fully connected route network to a strict single allocation hub route network when the passenger flow of each route in the fully connected route network increases by 14.9% year-on-year on the basis of the original value, under the driving force of the route network phase transition. This means that in the strict single allocation hub route network composed of nine city nodes, the city with the city number 1 is named Beijing, the city with the city number 2 is named Shanghai, and the city with the city number 3 is named Guangzhou, which are the three hub nodes [14].

For the already established route network, airlines always expect to complete corresponding target tasks as much as possible while maintaining the status quo when making local adjustments. That is, in order to solve the congestion problem of specific hub airports, direct flight routes can be opened to multiple non hub airport OD pairs passing through this hub airport. Each direct flight route opened will share a portion of the transportation flow of OD pairs and also cause changes in the route revenue in the network. In order to ensure the overall operational efficiency of the route network and optimize the structure of the route network during transportation operations, it is particularly important for airlines to determine whether it is necessary to open all OD to direct routes passing through this hub airport, or only open some OD to direct routes. The specific direct routes to be opened are particularly important. The more reasonable the evolution of the route network structure, the better the effectiveness of the route network operation.

2. Description and Mathematical Modeling of Mixed Airline Network Problems

In order to describe the emergence and evolution of hybrid airline networks, it is necessary to establish the following seven abstract relationship patterns:

Route Relationship Mode (Route Number, Route Mileage, Route Passenger Flow, Passenger Kilometer Revenue, Seat Kilometer Cost, First Transit Hub Airport Number, Second Transit Hub Airport Number)

Airport capacity relationship model (airport number, average number of seats, peak hour capacity)

Hub airport status relationship model (design capacity, peak hour passenger flow, congestion)

Open route relationship mode (route number, revenue change value caused by opening non hub airport direct flight routes, first transit hub airport number, passenger flow shared by opening a direct flight route for one transit, second transit hub airport number, passenger flow shared by opening a direct flight route for the second transit, whether the route has been opened)

Newly opened route relationship mode (route number, proportion of increased passenger flow on the route)

Among them, if the values of "first transit hub airport number" and "second transit hub airport number" in the "route relationship mode" are both equal to -1, it means that the route corresponding to the "route number" is a direct flight route; If the value of "first transit hub airport number" is not equal to -1, and the value of "second transit hub airport number" is equal to -1, it means that the route is a transit route; If the values of "first transit hub airport number" and "second transit hub airport number" are not equal to -1, it means that the route is a second transit route.

In the "Airport Capacity Relationship Model", the values of "Average Seating Capacity" and "Peak Capacity (Flights/Hours)" for non hub airports corresponding to "Airport Number" are both 0, indicating that only congestion at hub airports is considered.

The "design capacity" in the "hub airport status relationship model" refers to the maximum capacity carrying capacity of the hub airport. The design capacity of each hub airport can be calculated by multiplying the values of the "average seating capacity" and "peak capacity (flight/hour)" fields in the "airport capacity relationship model"; Whether there is congestion "is a 0-1 variable. When a hub airport is congested, the corresponding value of this attribute is 1, otherwise it is 0.

The "Route Opened" variable in the "Open Route Relationship Mode" is a 0-1 type variable. As the passenger flow of the route increases, if the route between non hub airports has been opened, the value of the "Route Opened" attribute is 1, otherwise it is 0. The main function of this relationship pattern is to store information on all possible routes that may be opened through hub airports.

The parameters involved in the evolution process of mixed route networks include: node set in the route network N ,$|N| = n$,hub airport node set P , $P \subset N, |P| = p$, airport nodes $i(i \in N)$, route mileage $D_{ij}(i, j \in n)$, and OD passenger flow $q_{ij}(i, j \in n)$. The passenger flow transported through direct routes is q_{ij}^{d}, the passenger flow transported through transit routes q_{ij}^{t}, the passenger kilometer revenue of the flight segment $r_{ij}(i, j \in n)$, the seat kilometer cost of the flight segment $c_{ij}(i, j \in n)$, the proportion of the original transit route passenger flow to OD passenger flow after opening the direct route on the original transit route α_1 , the proportion of the original transit route passenger flow to OD passenger flow after opening the direct route on the original secondary transit route α_2 , the design capacity of the hub airport $o_i(i \in P)$, and the peak passenger flow of the hub airport $u_i(i \in P)$.

The design capacity of hub airports $o_i(i \in P)$ is generally calculated by multiplying the maximum number of takeoffs (u_i) and landings during peak hours by the average number of aircraft seats (w_i). $o_i = w_i u_i$.

The peak hour passenger flow of hub airports $m_i(i \in P)$ is generally calculated by converting the annual passenger throughput of the airport into an average daily volume, and then considering the peak month coefficient of throughput and the proportion of peak hour passenger throughput to the total day passenger throughput. The conversion formula is equation (1):

$$m_i = e_i \times 1/365 \times g_i \times h_i \tag{1}$$

e_i is the annual passenger throughput of hub airport i, g_i is the peak month passenger throughput coefficient of hub airport i, and h_i is the proportion of peak hour passenger throughput of hub airport i to all day passengers.

Opening direct flights to non hub airports can cause changes in network revenue, and OD passengers will share the burden between direct and transit routes. Therefore, there is a difference between the network revenue after opening direct flights to non hub airports (R_1) and before opening (R_0), which may be positive or negative. However, for airlines, the purpose of opening direct flights is to ensure that passengers do not lose passengers due to network congestion and to maintain the stability of the original route network structure. Therefore, the goal of airlines (R) is to minimize the absolute value of the difference. $R= R_1- R_0$.

When an OD pair contains only one flight segment, its revenue is the number of OD passengers and the length of the flight segment (passenger kilometer revenue - seat kilometer cost). When an OD pair is transported by a transit route, its revenue is the sum of the revenue of each flight segment.

When a hub airport is congested but direct flights have not yet been added, the revenue corresponding to the routes passing through the hub airport (R_0) is equal to the product of "OD passenger flow", "segment mileage", and "segment revenue difference" in the "route relationship model", where "segment revenue difference" is the difference between "passenger kilometer revenue" and "seat kilometer cost" of the segment.

When a hub airport is congested and non hub airport direct flights are added, the passenger flow of the original OD pair will have two transportation paths: one is the direct flight path between non hub airports, and the other is the transit path through hub airports. At this time, the route revenue generated by the OD pair (R_1) is the sum of the direct flight revenue and the transit route revenue.

In the airline network, congestion in nodes is mainly concentrated at hub airports, so the main consideration is to ensure that the passenger throughput of hub airports during peak hours does not exceed their peak hour capacity. The peak hour passenger flow at the hub airport is equal to the sum of all OD passenger flows passing through the hub airport.

The corresponding mathematical model can be represented by equations (2) to (4).

$$\min R \tag{2}$$

s.t.

$$q_{ij}^{d}+q_{ij}^{t}=q_{ij} \tag{3}$$

$$m_k < o_k, k \in P \tag{4}$$

Equation (2) represents the airline's pursuit of network structural stability when adjusting the route network after the formation of the full hub route network, while Equation (3) represents the sharing of OD passenger traffic between direct and transit routes; Equation (4) indicates that hub airports will not experience peak hour traffic exceeding their design capacity.

3. Steps for Solving the Hybrid Airline Network Model

Step 1: Determine the design capacity of each hub airport;

Step 2: Determine the passenger flow of each OD after the given growth rate;

Step 3: Determine the peak flow of each OD flow through each hub airport;

Step 4: Compare whether the peak flow of each hub airport exceeds the design capacity of each hub airport. If "yes", proceed to Step 2; if "no", proceed to Step 5;

Step 5: Determine the transfer routes for each OD passenger flow passing through the hub airport. If there is no direct flight route for the transfer route, calculate the passenger flow q_{ij}^{d} shared with the direct flight route after adding a direct flight route to the transfer route;

Step 6: Establish the following integer linear programming and call the integer linear programming solving module to solve it, which can be represented by equations (5) and (6);

$$\min Z=\sum b_{ij}x_{ij} \tag{5}$$

$$Aq_{ij}^{d}x_{ij} \geq m_k - o_k \qquad k \in P \tag{6}$$

Among them, there is a 0-1 variable x_{ij}. When 1 is taken, it indicates the addition of a direct flight route in the OD passenger flow ij, and when 0 is taken, it indicates that no direct flight route is opened in the OD passenger flow ij. The coefficient matrix representing the OD passenger flow b_{ij} after adding a direct flight route to the original transfer route is a 0-1 matrix, where the coefficient matrix is taken as 1 when the transfer route passes through the hub point k, otherwise it is taken as 0.

Step 7: Save the newly opened direct flight route, modify the corresponding OD passenger flow q_{ij} ($q_{ij}=q_{ij}-q_{ij}^{d}$), and proceed to Step 2.

4. Case Study on the Evolution of Hybrid Airline Network

The example will still use the route network composed of 9 city nodes in Xu Shunzhi et al. (2014) to analyze how the strict single allocation hub route network evolves further under the assumption of year-on-year growth in passenger flow for different types of routes. Referring to the algorithm steps and possible parameter variables in the rule constraints mentioned above, it is assumed that the passenger flow of a transfer path accounts for the proportion of OD to passenger flow when opening a direct flight route between non hub airports; The proportion of passenger flow on the secondary transfer path to the OD passenger flow when opening direct flights between non hub airports; The "average seating capacity" and "peak hour capacity" of the hub airport "Beijing" are 180 and 58 flights per hour, respectively; The "average seating capacity" and "peak hour capacity" of the hub airport "Shanghai" are 180 and 50 flights per hour, respectively; The "average seating capacity" and "peak hour capacity" of the hub airport "Guangzhou" are 180 and 50 * flights per hour, respectively (*, the capital, Guangzhou, and Shanghai are all international airports with a large number of international flights, so the capacity of international flights is deducted); The proportion of peak hour flights at hub airports to all day flights is 9%; The peak hour passenger flow at hub airports has an expansion ratio (peak month coefficient) of 1.1 compared to the annual average passenger flow on air routes; The specific information of the "route relationship mode" when strictly allocating hub route networks is shown in Table 1.

Using Matlab 7.8.0 (R2009a) to program the mathematical model for the formation and evolution of mixed networks according to the specified algorithm solving steps, the results of

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the "newly opened route relationship mode" after all parameter values are determined are shown in Table 2.

Table 1. Route Information Table for Strictly Single Allocation Hub Route Network

Route name	Route mileage	Airline passenger flow	Revenue per passenger kilometer	Seat kilometer cost	Transfer Airport 1 Number	Transfer Airport 2 Number
12	1178	6635736	0.680786	0.517388	-1	-1
13	1967	3031864	0.743486	0.50821	-1	-1
14	412	367025	1.081993	0.792339	-1	-1
15	646	1125709	0.723549	0.557338	-1	-1
16	1133	1289532	0.614802	0.478089	2	-1
17	1500	822477	0.578727	0.46348	2	-1
18	2266	1818255	0.581259	0.450104	3	-1
19	2710	964230	0.657494	0.456854	3	-1
23	1308	2975483	0.657494	0.456854	-1	-1
24	852	690319	0.5239	0.501282	1	-1
25	693	1641127	0.743486	0.50821	1	-1
26	761	1021363	0.632248	0.465418	-1	-1
27	418	1083203	0.595396	0.446715	-1	-1
28	2042	768911	0.568851	0.406164	3	-1
29	2050	1185279	0.68686	0.489958	3	-1
34	1664	537169	0.601133	0.434463	1	-1
35	1867	422839	0.61251	0.434031	1	-1
36	873	851365	0.644457	0.486403	2	-1
37	1015	842231	0.598871	0.476603	2	-1
38	1357	746278	0.586925	0.473239	-1	-1
39	747	1233476	0.519086	0.436048	-1	-1
45	315	73200	0.644457	0.486403	1	-1
46	890	182701	0.5239	0.501282	1	2
47	1170	24975	0.5239	0.501282	1	2
48	2080	129732	0.5239	0.406342	1	3
49	2497	24975	0.5239	0.501282	1	3
56	1017	322878	0.617707	0.490069	1	2
57	1102	51601	0.5239	0.501282	1	2
58	2373	40688	0.561673	0.390155	1	3
59	2854	40688	0.561673	0.486403	1	3
67	750	182222	0.561673	0.490069	2	-1
68	1364	591503	0.529099	0.406342	2	3
69	1700	249588	0.529099	0.490069	2	3
78	1926	179684	0.617707	0.486403	2	3
79	1802	51237	0.617707	0.490069	2	3
89	1213	179254	0.5239	0.486403	3	-1

According to Table 2, in a strict single allocation hub route network, when the passenger flow of different types of routes increases by less than 54%, none of the three hub airports experience congestion, and the route network structure will still be a strict single allocation hub route network, with only 9 direct flight segments in the network; When the passenger flow of different types of routes increases to 54%, the numbers of direct flights between airport nodes that need to be opened are 67 and 79, and the number of direct flight segments in the network is 11; When the passenger flow of different types of routes increases to 71%, the number of direct flight routes between airport nodes that need to be opened is 47, and the number of direct flight segments in the network increases to 12; When the passenger flow of the route continues to increase to 101%, no new direct flights will be opened.

Table 2. Newly Opened Non Hub Airport Direct Flight Routes

Route number	The proportion of increased passenger flow on air routes
67	54%
79	54%
47	71%
17	79%
45	87%
58	87%
57	95%
46	97%
49	98%
59	98%
56	99%
48	101%

But when the traffic in the network continues to increase to 109%, the network reaches saturation, meaning that whether or not direct flights are added, congestion at hub airports cannot be avoided. This is because the OD passenger flow between hub nodes has exceeded the design capacity of the hub airport. No matter how much passenger traffic increases in the network, there will not be 36 direct flight segments. That is to say, the strict single allocation hub route network cannot transform into a fully connected route network under the same year-on-year growth of passenger traffic on different types of routes. This is different from the result of differential growth in passenger traffic on different types of routes and the strict single allocation hub route network transforming into a fully connected route network when the passenger traffic is large enough.

The order and quantity of increase in direct flight routes are basically the same for different OD flow passenger transfer ratios, as shown in Table 3.

Table 3. Increased Direct Airline Routes under Different OD Flow Passenger Transfer Ratios

$\alpha_1 = 0.5$ $\alpha_2 = 0.4$		$\alpha_1 = 0.4$ $\alpha_2 = 0.3$		$\alpha_1 = 0.3$ $\alpha_2 = 0.2$		$\alpha_1 = 0.2$ $\alpha_2 = 0.1$	
Route number	Proportion of increase in OD passenger traffic	Route number	Proportion of increase in OD passenger traffic	Route number	Proportion of increase in OD passenger traffic	Route number	Proportion of increase in OD passenger traffic
67	54%	67	54%	67	54%	67	54%
79	54%	79	62%	79	62%	78	63%
47	71%	17	72%	17	72%	47	73%
17	79%	78	84%	47	85%	79	73%
45	87%	45	86%	78	85%	45	91%
58	87%	47	92%	45	91%	58	91%
57	95%	57	92%	57	91%	57	93%
46	97%	58	92%	58	91%	49	102%
49	98%	59	92%	46	96%	59	102%
59	98%	48	97%	59	97%	48	103%
56	99%	46	98%	48	98%	46	104%
48	101%	56	99%	56	99%	56	106%
		24	102%	24	102%	24	109%
		25	107%	25	107%	17	114%
		16	108%	16	108%	25	115%
						16	116%

From Table 3, it can be seen that the higher the proportion of transit passengers in the OD passenger flow, the fewer direct flight routes are opened. From Chapter 3, it can be seen that in the case of a large OD passenger flow, the economy of a strict hub route network is superior to that of a fully connected route network. However, in order to ensure the high speed of air transportation, it is necessary to establish certain direct flight routes, which reduces the economic benefits of airlines. With the growth of OD traffic, more direct flight routes need to be opened, which is also an inevitable choice for airlines to face competition from other modes of transportation.

5. Conclusions

Through the study of the formation and

evolution of mixed route networks, it was found that fully connected route networks and strictly single allocation hub route networks are two ideal states in route network construction. In reality, the route network structure is mostly a mixed route network, which includes both hub and non hub airports, direct routes between hub and hub airports, and direct routes between hub and non hub airports. The evolution of hybrid airline networks should not only focus on the congestion situation of hub airports, but also consider the changes in passenger flow on different routes in the network. That is, when passenger flow increases within a certain proportion range, OD can be shared among some of the transported passenger flow by opening direct flight routes between airport nodes, reducing the transfer traffic passing through specific hub airports and alleviating the congestion level of hub airports; When the passenger flow increases beyond a certain range, even if new direct flight routes are opened in the network, it cannot solve the problems of network congestion, flight delays, etc., because the actual passenger flow passing through the hub airport has exceeded its maximum design capacity at this time; When passenger traffic continues to grow, only by increasing the design capacity of hub airports or expanding more hub airports can the overall healthy and orderly operation of the mixed route network be ensured.

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