Analysis of Dynamic Force Error in Fatigue Testing under Nonlinear Mechanical Model Based on Finite Element Numerical Simulation

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Abstract: This article summarizes the problems existing in multi degree of freedom linear vibration systems and finds that it is necessary to consider the influence of structural nonlinear sources on measurement results in some cases. Therefore, the perturbation method is used to theoretically analyze the nonlinear response caused by subharmonics under nonlinear characteristics. The influence of even and odd harmonics on displacement waveforms is theoretically derived. At the same time, finite element numerical simulation method is used to analyze the transient response of a two degree of freedom nonlinear stiffness spring mass vibration system. It is found that nonlinear sources will cause high-frequency vibration of subharmonics in the vibration system and have a more significant impact on acceleration measurement results. This situation will have an impact on the process of correcting dynamic force measurement errors using the A1.4 accelerometer method according to ASTM E467-2008D.

Keywords: Finite Element; Nonlinear; Dynamic Force; Fatigue Testing Machine

1. Introduction

Under the action of alternating stress, due to the presence of cracks or microcracks within the material, stress concentration effects will form at the crack tip. When the accumulated energy at the crack tip exceeds the material's fracture toughness, the crack will enter the stage of unstable propagation. Thus, material fracture may occur after a certain number of loading cycles. In order to prevent fatigue failure accidents in fields such as machinery, aerospace, civil engineering, and hydraulic engineering, professional laboratories often conduct research on the fatigue performance of materials. Especially for aviation materials, fatigue testing machines have become very important experimental equipment in aviation material testing for the calendar life determination and extension needs of newly developed and existing aircraft[1].

In terms of theoretical derivation of dynamic forces in fatigue testing, Guangzhen established three mechanical models based on three different structural types of fatigue testing machines, proving that there are differences in the forces between the fatigue testing machine and the actual tested piece, and found that it is caused by the inertia mass of the connection between the centers of mass of the two[2-4]. Sun Lei established a dynamic model of the fatigue testing machine and solved the amplitude of the vibration system through the motion differential equation. The amplitude frequency characteristics of the vibration system were obtained through simulation analysis[5]. Wang Meng from Xiamen University established a three degree of freedom spring mass vibration mechanics model for a fatigue testing machine, and studied the influence of the stiffness of the tested sample on the natural frequency of the system, and derived a theoretical derivation formula for dynamic force error [6].

In recent years, in-depth research has been conducted on the dynamic force calibration of fatigue testing machines both domestically and internationally. However, there are still nonlinear problems in the actual fatigue testing conditions. Simply using linear vibration theory for research is not comprehensive enough. Therefore, this article will conduct finite element numerical simulation analysis on the nonlinear problems in actual fatigue testing or calibration testing to analyze the errors of dynamic forces.

2. Problems in the Theoretical Model of Fatigue Testing Machine under Multi Degree of Freedom Linear Vibration System

At present, the vast majority of theoretical analysis of fatigue testing machines in the industry is based on the analysis of linear vibration systems with multiple degrees of freedom. In engineering practice, the main method is to establish vibration differential equations based on the dynamic balance method of the d'Alembert principle[7-9]. However, when facing practical problems, due to the infinite degree of freedom distributed vibration mass system of the structure and the possibility of large displacement, large rotation, initial stiffness change of the structure, non ideal linear materials, mechanical structure clearance and frictional vibration in dynamic problems, such as aeroelastic analysis of wing large deformation in the aerospace field and structural elastoplastic response analysis in the civil engineering field, nonlinear problems often lead to very complex dynamic analysis of structures in practical engineering applications.

Especially in electromagnetic resonance fatigue testing machines, when the natural frequency of the fatigue testing machine test piece system is close to the excitation frequency, small amplitude nonlinear stiffness may cause subharmonic vibration phenomenon during fatigue testing or calibration testing. It is generally believed that the presence of quadratic and cubic nonlinear terms in the vibration control equation is the reason for the generation of second and third harmonic vibrations in the system. The nonlinear sources in the working state of the fatigue testing machine mainly include geometric nonlinearity and material nonlinearity. Because during the process of applying dynamic forces to the tested sample or force calibration device, the structure may experience nonlinear responses due to geometric (gap) factors, and fatigue testing machine fixtures, connectors, etc. may exhibit nonlinear responses due to the nonlinear stress-strain relationship of the material itself, which is not a completely ideal linear relationship. Meanwhile, exposure to nonlinearity can lead to complex vibration collision dynamics problems in vibration systems, resulting in nonlinear behaviors such as harmonics. This article will conduct finite element simulation on the superharmonic phenomenon caused by the nonlinear characteristics of materials in the calibration simulation test of fatigue testing machine, and obtain the vibration spectrum and loading waveform.

3. Analysis of Nonlinear Response Caused by Superharmonics under Nonlinear Characteristics

Through structural tensile stiffness testing of a 500kN electromagnetic resonance fatigue testing machine, it was found that the tensile displacement load curve of the fatigue testing machine is not a completely ideal linear relationship. Due to the nonlinear stiffness of the structure, the second and third terms can lead to the generation of superharmonic waves. Its essence is the redistribution of energy in the frequency domain by the solution of nonlinear differential equations. Nonlinear effects cause displacement waveform responses to deviate from pure sine waves, especially manifested as the superposition of multiple harmonics in acceleration waveform responses.

Consider the nonlinear stiffness term of the motion equation for a single degree of freedom system consisting of a fixture, link, and base under a fatigue testing machine

$$m\ddot{x} + c\dot{x} + k_1 x + k_2 x^2 + k_3 x^3 = F_0 \cos(\omega t)$$

Among them, k_1 is the linear stiffness, and k_2 and k_3 are the quadratic and cubic coefficients of the stiffness, respectively.

Using perturbation method, assuming that the steady-state displacement response can be expressed as a series expansion of the small parameter ε

$$x(t) = \sum_{i=1}^{n} \varepsilon^i x_i(t)$$

Expand the equation of motion to the power of ε, assuming there is $k_2 = \varepsilon\alpha$ 、 $k_3 = \varepsilon\beta$

The linear solution of the equation of motion is

$$mx_1 + cx_1 + k_1 x_1 = F_0 \cos(\omega t)$$

The steady-state solution can be expressed as

$$x_1(t) = A\cos(\omega t + \varphi)$$

Substituting $x_1(t)$ into the quadratic nonlinear term $k_2 x^2$, there is

$$x_1^2 = A^2 \cos^2(\omega t + \varphi) = \frac{A^2}{2}[1 + \cos(2\omega t + 2\varphi)]$$

Therefore, the first-order equation is

$$\mathrm{m}\ddot{x}_2 + c\dot{x}_2 + k_1 x_2 = -\alpha A^2 \cos^2(\omega t + \varphi) = -\frac{\alpha A^2}{2}[1 + \cos(2\omega t + 2\varphi)]$$

The right-hand side of the equation contains a DC component, which will cause static displacement, while the second harmonic excitation part will generate a displacement response $x_2(t)$ with a frequency of 2ω in the forced vibration system. Since the second harmonic is caused by an asymmetric elastic restoring force $k_2 x^2$, the displacement response of the vibration system will exhibit asymmetric distortion (asymmetric displacement waveform up and down).

Substituting $x_1(t)$ into the cubic nonlinear term $k_3 x^3$, there is

$$x_1^3 = A^3\cos^3(\omega t+\varphi) = \frac{A^3}{4}[3\cos(\omega t+\varphi)+\cos(3\omega t+3\varphi)]$$

Therefore, the second-order equation is

$$m\ddot{x}_3 + c\dot{x}_3 + k_1 x_3 = -\beta x_1^3 - 2\alpha x_1^3 - 2\alpha x_1 x_2$$

$$-\beta x_1^3 = -\frac{\beta A^3}{4}[3\cos(\omega t+\varphi)+\cos(3\omega t+3\varphi)]$$

The right-hand side of the equation contains a fundamental frequency modulation term, which will correct the amplitude of the fundamental wave. The third harmonic excitation part will generate a displacement response $x_3(t)$ with a frequency of 3ω in the forced vibration system. Since the third harmonic is caused by a symmetric nonlinear elastic restoring force $k_3 x^3$, the displacement response of the vibration system will exhibit symmetry hardening or softening (the displacement waveform will appear as a "peak" or "flattening").

Since acceleration is the second derivative of displacement, there is

$$x(t)'' = -\omega^2 x(t)$$

The frequency spectrum of acceleration is equivalent to the displacement spectrum multiplied by $-\omega^2$. Therefore, the frequency of subharmonics will amplify the amplitude of acceleration. The higher the frequency of subharmonics, the more significant the amplification effect of acceleration amplitude.

4. Displacement and Acceleration Response Analysis of Nonlinear Vibration System based on Finite Element Numerical Simulation

This article takes the 500kN resonant fatigue testing machine as an example to numerically simulate the load condition with an average cyclic force of 100kN, a cyclic force amplitude of 50kN, and a load frequency of 140.5Hz. The entire system is simplified into a two degree of freedom spring mass vibration model, and the entire load step is divided into a static loading stage and a transient response stage. The entire process is set as a transient analysis type. In the transient response stage, in order to avoid the influence of free vibration, the analysis time is set to 20 seconds to obtain the response results entering the steady-state vibration stage. In order to ensure the calculation accuracy of the finite element model and avoid non convergence, the time for each load sub step is set to 0.0001s. The type of applied load is selected as step load, and the sine wave is input into the finite element model through the formula editor. The stiffness of the spring mass vibration system is input into the finite element model based on actual measurement results, and its nonlinear stiffness curve is shown in Figure 1.

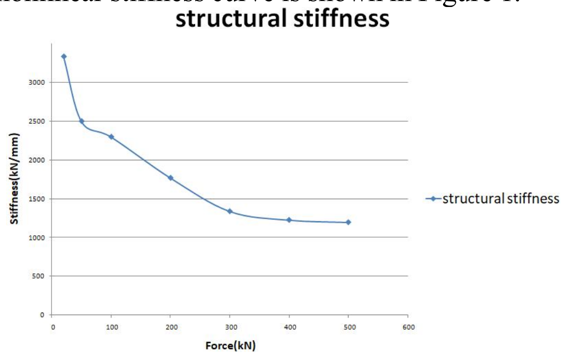

Figure 1. Nonlinear Stiffness of Structure

By observing the acceleration waveform, it can be seen that the acceleration waveform clearly has waveform characteristics caused by even harmonics and odd harmonics. Due to the influence of even harmonics, the acceleration waveform shows asymmetry in the upper and lower parts. At the same time, due to the influence of odd harmonics, the acceleration waveform shows characteristics of peak shaving at the top and sharpness at the bottom. In the presence of nonlinear characteristics in the stiffness of the vibration system, the finite element calculation results are consistent with the theoretical analysis, and there will be subharmonic high-frequency vibration in the vibration system, which will have a more significant impact on the acceleration measurement results.

According to ASTM E467-2008D, in the typical dynamic force calibration process of fatigue testing machines, the dynamic force error mainly comes from the mass acceleration between the load sensor and the sample (or metrological calibration device). According to the A1.4 error calculation method of ASTM E467-2008D, this specification recommends that the accelerometer method be preferred for correcting dynamic force measurement errors. However, according to the finite element analysis in this article, when there is a nonlinear source in the structure, the acceleration measurement results are simultaneously composed of the acceleration response of the fundamental wave and the harmonic wave. High frequency harmonic waves will have an impact on the acceleration measurement, leading to errors in the inertial force correction of the dynamic force calibration results.

5. Conclusion

This article theoretically derives the effects of even and odd harmonics on displacement waveforms caused by subharmonics under nonlinear characteristics through perturbation method. Since acceleration is the second derivative of displacement, the frequency spectrum of acceleration is equivalent to multiplying the displacement spectrum by $-\omega^2$. Therefore, the frequency of subharmonics will amplify the amplitude of acceleration. The higher the frequency of subharmonics, the more significant the amplification effect of acceleration amplitude.

On the basis of theoretical derivation, this paper conducted transient response analysis of a two degree of freedom spring mass vibration system by establishing finite element numerical simulation, and obtained displacement response time history curves and acceleration response time history curves under structural nonlinear stiffness. By observing the waveform, a conclusion consistent with theoretical analysis was obtained that nonlinear sources will exhibit subharmonic high-frequency vibrations in the vibration system, and will have a more significant impact on acceleration measurement results. This situation will have an impact on the process of correcting dynamic force measurement errors using the A1.4 accelerometer method according to ASTM E467-2008D.

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